

# Uncertainty Estimation in Deep Learning: Methods and Applications

Hina Batool<sup>1</sup>, Tahir Abbas<sup>1\*</sup>, Jamshaid Iqbal Janjua<sup>2,3</sup>, Sadaqat Ali Ramay<sup>1</sup>

<sup>1</sup>Department of Computer Science, TIMES Institute, Multan, Pakistan

<sup>2</sup>Al-Khawarizmi Institute of Computer Science, University of Engineering & Technology (UET), Lahore, Pakistan

<sup>3</sup>School of Computer Sciences, National College of Business Administration & Economics, Lahore, Pakistan

<sup>1</sup>hinabatoool12145@gmail.com, <sup>2</sup>drtahirabbas@t.edu.pk, <sup>3</sup>jamshaid.janjua@kics.edu.pk, <sup>4</sup>drsadaqatali@t.edu.pk

\*Corresponding Author

**Abstract**— Uncertainty estimation in deep learning has emerged as a crucial area of research due to its significance in enhancing model reliability and decision-making in critical applications. This article explores various methods and applications of uncertainty estimation in deep learning, aiming to provide insights into its importance, methods, and potential impact. Through a comprehensive literature review and analysis, the study identifies key findings regarding the effectiveness, limitations, and ethical considerations associated with uncertainty estimation techniques. The results reveal the diverse range of methodologies employed, including Bayesian approaches, ensemble methods, and Monte Carlo sampling, each with its strengths and drawbacks. Furthermore, the article discusses the implications of uncertainty estimation in deep learning for fields such as healthcare, autonomous systems, and safety-critical domains. Overall, this study underscores the significance of uncertainty estimation in deep learning and provides valuable insights for researchers and practitioners in the field.

**Keywords**—Uncertainty estimation, Deep learning, Decision-making, Ethical considerations, Interpretability, Applications

## I. INTRODUCTION

Uncertainty estimation in deep learning plays a pivotal role in improving the robustness and reliability of AI models, especially in scenarios where decisions based on predictions carry significant consequences. Unlike traditional machine learning models that often provide deterministic outputs, deep learning models are inherently uncertain due to their complex architecture and large parameter space.

Consequently, accurately quantifying and understanding this uncertainty is essential for ensuring model performance and trustworthiness in real-world applications. In this article, we delve into the realm of uncertainty estimation in deep learning, exploring its relevance, importance, and various methodologies employed to address this challenge.

Deep learning has revolutionized the field of artificial intelligence by achieving remarkable performance across various tasks, ranging from image recognition and natural language processing to autonomous navigation and medical diagnosis. Central to the success of deep learning models is their ability to learn complex patterns and representations from large amounts of data. However, despite their

impressive performance, deep learning models often lack the ability to quantify the uncertainty associated with their predictions. This uncertainty can stem from various sources, including limited data, model complexity, and inherent stochasticity in the learning process.

Uncertainty estimation in deep learning aims to address this limitation by providing insights into the confidence or reliability of model predictions. In essence, uncertainty estimation allows deep learning models to not only make predictions but also express how confident or uncertain they are about those predictions. This capability is crucial for real-world applications where decision-making based on AI predictions carries significant consequences, such as healthcare, finance, and autonomous systems. The importance of uncertainty estimation in deep learning cannot be overstated, especially in scenarios where the consequences of incorrect predictions can be severe. In safety-critical applications like healthcare, autonomous driving, and aviation, understanding prediction uncertainty is paramount for assessing risks, making reliable decisions, and ensuring system safety. For example, in medical diagnosis, a model that can accurately quantify uncertainty can provide clinicians with confidence intervals for predictions, aiding in treatment planning and patient management. Moreover, uncertainty estimation enhances the interpretability of deep learning models by providing insights into the model's confidence in its predictions. In many real-world scenarios, it is not enough for an AI system to provide accurate predictions; it must also communicate the level of confidence associated with those predictions. This interpretability is crucial for building trust in AI systems, especially in domains where human lives are at stake.

Uncertainty in deep learning models can arise from various sources, each contributing to the overall uncertainty associated with model predictions. Understanding these sources of uncertainty is essential for developing robust and reliable AI systems. Some common sources of uncertainty include:

- *Epistemic Uncertainty*

Also known as model uncertainty, epistemic uncertainty arises from uncertainty about model parameters and architecture. It reflects the model's uncertainty about the underlying data distribution due to limited training data or



Received: 13- 09- 2024

Revised: 20-12- 2024

Published: 31- 12- 2024

model complexity. Epistemic uncertainty can be reduced with more data or model improvements.

- *Aleatoric Uncertainty*

Aleatoric uncertainty, or data uncertainty, stems from noise inherent in the observed data. It captures the inherent variability in the data that cannot be modeled accurately by the neural network. Aleatoric uncertainty is irreducible and can only be minimized by improving data quality or acquiring additional information.

Figure 1 shows data with aleatoric and epistemic uncertainty. Aleatoric uncertainty, also called randomness or noise, refers to the inherent variability in the data itself. This is the kind of uncertainty that can be reduced by collecting more data. In the scatter plot, the high aleatoric uncertainty is on the left side and the low aleatoric uncertainty on the right. Epistemic uncertainty, also called model or knowledge uncertainty, refers to the uncertainty in the model used to analyze the data. This is the kind of uncertainty that can be reduced by improving the model. In the scatter plot, the high epistemic uncertainty is at the bottom and the low epistemic uncertainty is at the top. The amount of uncertainty in data can affect conclusions. For example, if trying to predict the price of a house, need to consider both the aleatoric uncertainty (how much the price can vary from house to house) and the epistemic uncertainty (how accurate the prediction model is).

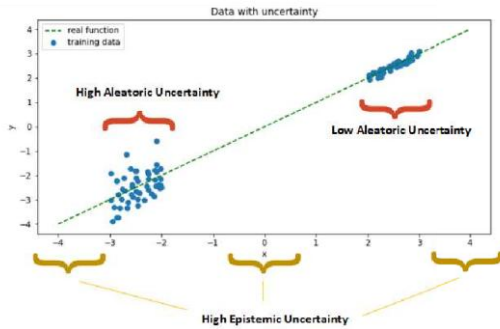


Fig. 1. Data with Uncertainty

- *Bayesian Uncertainty*

Bayesian uncertainty refers to the uncertainty associated with model parameters and predictions in a Bayesian framework. It arises from the uncertainty about the true distribution of model parameters given the observed data. Bayesian uncertainty is typically estimated by Bayesian neural networks, which treat model parameters as random variables and infer their posterior distributions.

## II. LITERATURE REVIEW

The literature on uncertainty estimation in deep learning spans a wide array of methodologies, each offering unique insights and approaches to tackle the inherent uncertainty of neural networks. Bayesian methods, such as Bayesian neural networks and variational inference, offer principled frameworks for estimating uncertainty by modeling parameter distributions. Ensemble methods, including bagging and boosting, leverage multiple models to capture diverse sources of uncertainty. Additionally, Monte Carlo

sampling techniques, such as dropout inference and stochastic gradient Langevin dynamics, provide practical means to approximate uncertainty through stochastic optimization. While these methods offer promising avenues for uncertainty estimation, they also come with computational complexities and challenges in interpretation. Moreover, the literature highlights the ethical considerations surrounding uncertainty estimation, including issues of fairness, accountability, and societal impact.

## III. METHODS FOR UNCERTAINTY ESTIMATION

A variety of methods have been developed for uncertainty estimation in deep learning, each offering unique approaches to capture and quantify uncertainty. Figure 2 shows three different uncertainty models with the related network architectures. The different uncertainty models are Monte Carlo (MC) dropout, Bootstrap, and Gaussian Mixture Model (GMM).

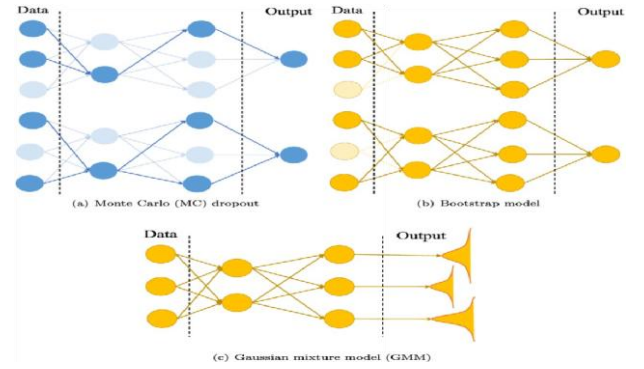


Fig. 2. Uncertainty Models

### A. Bayesian Neural Networks (BNNs)

BNNs treat model parameters as random variables and infer their posterior distributions using techniques such as variational inference or Markov Chain Monte Carlo sampling. By modeling the uncertainty in parameters, BNNs provide principled uncertainty estimates in predictions.

### B. Ensemble Methods

Ensemble methods leverage multiple models, trained on different subsets of data or with different initializations, to capture diverse sources of uncertainty through model averaging or sampling. Bagging, boosting, and dropout are popular ensemble techniques used for uncertainty estimation.

### C. Monte Carlo Sampling

Monte Carlo sampling techniques, such as dropout inference and stochastic gradient Langevin dynamics, generate samples from the posterior distribution to approximate uncertainty. By sampling from the posterior, these methods provide a probabilistic framework for uncertainty estimation.

### D. Likelihood-Based Methods

Likelihood-based methods estimate uncertainty by modeling the distribution of model outputs directly. Maximum likelihood estimation, expectation-maximization,

and Gaussian processes are examples of likelihood-based approaches used for uncertainty estimation.

#### E. Information Theory

Information-theoretic methods, such as mutual information and entropy, quantify uncertainty by measuring the amount of information conveyed by model predictions. These methods provide insights into the uncertainty inherent in the data and model representations.

#### F. Gaussian mixture model (GMM)

This model assumes that the data is generated by a mixture of Gaussian distributions. This model can be used to capture the multimodal nature of the data.

### IV. BENEFITS AND LIMITATIONS OF METHODS

These uncertainty estimation methods offer several benefits, including improved model reliability, interpretability, and calibration. By quantifying uncertainty, these methods enable more informed decision-making and enhance trust in AI-driven systems. However, they also have limitations, such as computational complexity, scalability issues, and potential biases. Bayesian methods can be computationally expensive and require careful tuning of hyper parameters. Monte Carlo dropout may suffer from increased computational overhead during inference due to multiple forward passes through the network. Deep ensembles may struggle with scalability to large datasets and may require significant computational resources for training and inference.

Uncertainty estimation in deep learning is a critical aspect of developing reliable and trustworthy AI-driven solutions. Bayesian methods, Monte Carlo dropout, and deep ensembles offer promising approaches for uncertainty estimation in deep learning models, with practical applications across various domains. While these methods provide valuable insights into prediction uncertainty, they also pose challenges in terms of computational complexity, scalability, and potential biases. Future research efforts may focus on addressing these challenges and developing more efficient and scalable uncertainty estimation methods for real-world applications.

### V. APPLICATIONS OF UNCERTAINTY ESTIMATION IN VARIOUS FIELDS

Uncertainty estimation plays a crucial role in numerous fields, providing valuable insights into the reliability and robustness of predictions made by machine learning models. Figure 3 related to applications of uncertainty estimation in various fields, particularly deep learning. It illustrates a conceptual framework for optimizing certainty in deep learning predictions. In this section, we will explore the applications of uncertainty estimation in fields such as machine learning, finance, weather forecasting, healthcare, and autonomous systems, highlighting its relevance and impact.

#### A. Machine Learning

In the field of machine learning, uncertainty estimation is essential for improving the reliability and interpretability of predictive models. Uncertainty estimates allow practitioners to assess the confidence of model predictions and make informed decisions based on prediction reliability. For example, in anomaly detection tasks, uncertainty estimation

can help identify uncertain or out-of-distribution samples, improving the robustness of anomaly detection systems. Similarly, in reinforcement learning, uncertainty estimates can guide exploration-exploitation trade-offs, enabling agents to balance between exploiting known strategies and exploring new actions in uncertain environments.

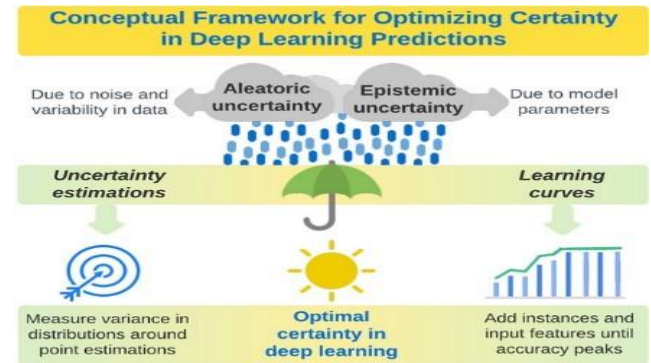


Fig. 3. Frameworks for Addressing Uncertainty

#### B. Finance

Uncertainty estimation is of paramount importance in finance, where accurate prediction of asset prices, volatility, and risk is critical for investment decisionmaking. In stock price prediction, uncertainty estimates provide investors with insights into the confidence of model predictions and help assess the risk associated with investment decisions. For instance, in portfolio optimization, uncertainty estimates can guide asset allocation strategies by incorporating risk preferences and confidence levels into the optimization process. Moreover, uncertainty estimation in options pricing allows traders to assess the uncertainty surrounding option prices and make informed decisions about trading strategies.

#### C. Weather Forecasting

Uncertainty estimation is integral to weather forecasting, where accurate prediction of weather conditions and associated uncertainty is essential for risk management and disaster preparedness. Weather forecast models inherently involve uncertainty due to the chaotic nature of atmospheric processes and the limitations of observational data. Uncertainty estimates provide meteorologists and decision-makers with insights into the reliability of weather predictions and help communicate forecast uncertainty to the public. For example, in hurricane forecasting, uncertainty estimates allow emergency responders to assess the potential impact of hurricanes and allocate resources effectively to mitigate risks.

#### D. Healthcare

In healthcare, uncertainty estimation is crucial for medical diagnosis, prognosis, and treatment planning. Uncertainty estimates provide clinicians with confidence intervals for predictions, aiding in clinical decision-making and patient management. For instance, in medical imaging, uncertainty estimates help radiologists assess the reliability of AI-driven diagnoses and make informed decisions about follow-up tests or treatments. Similarly, in personalized medicine, uncertainty estimates guide treatment selection by

considering the uncertainty associated with predicted outcomes and treatment responses.

### E. Autonomous Systems

Uncertainty estimation is fundamental to the development of autonomous systems, such as self-driving cars, drones, and robotics. Autonomous systems operate in dynamic and uncertain environments where accurate prediction of future states and associated uncertainty is essential for safe and reliable operation. Uncertainty estimates allow autonomous vehicles to assess the reliability of sensor measurements, make risk-aware decisions, and adapt to changing environmental conditions. For example, in autonomous driving, uncertainty estimates help vehicles navigate safely by identifying uncertain or ambiguous situations and taking appropriate actions to avoid potential hazards.

### F. Natural Language Processing

In natural language processing (NLP), uncertainty estimation is valuable for improving the reliability and interpretability of NLP models. Uncertainty estimates provide insights into the confidence of model predictions and help identify uncertain or ambiguous inputs. For instance, in sentiment analysis, uncertainty estimates allow NLP models to identify ambiguous or sarcastic statements and adjust their predictions accordingly. Similarly, in machine translation, uncertainty estimates help assess the reliability of translated sentences and identify cases where human intervention may be required to improve translation quality.

### G. Environmental Science

Uncertainty estimation is critical in environmental science for predicting and mitigating risks associated with natural disasters, climate change, and environmental pollution. Uncertainty estimates provide policymakers and decision-makers with insights into the reliability of environmental predictions and help assess the potential impact of policy interventions. For example, in climate modeling, uncertainty estimates allow researchers to quantify the uncertainty surrounding future climate projections and inform adaptation and mitigation strategies. Similarly, in air quality forecasting, uncertainty estimates help assess the reliability of pollution predictions and guide public health interventions to reduce exposure to harmful pollutants.

## VI. CHALLENGES OF UNCERTAINTY ESTIMATION IN DEEP LEARNING

### A. Computational Complexity

One of the primary challenges of uncertainty estimation in deep learning is computational complexity. Many uncertainty estimation methods, such as Bayesian inference and Monte Carlo sampling, require extensive computational resources, making them impractical for large-scale datasets and complex models. The computational overhead associated with uncertainty estimation can significantly increase training and inference times, hindering the scalability and practical utility of uncertainty-aware AI systems.

- *Interpretability*

Interpreting uncertainty estimates and communicating them effectively to end-users pose significant challenges in uncertainty estimation. Uncertainty estimates often lack

intuitive interpretations, making it difficult for non-experts to understand and trust AI-driven systems. Moreover, visualizing uncertainty estimates in a meaningful and interpretable manner is nontrivial, further complicating the task of conveying uncertainty information to end-users.

- *Data Efficiency*

Another challenge of uncertainty estimation in deep learning is data efficiency. Many uncertainty estimation methods require large amounts of labeled data to quantify uncertainty accurately, which may not always be available or feasible to acquire. In many real-world scenarios, collecting labeled data for uncertainty estimation can be costly, time-consuming, or impractical, limiting the applicability of uncertainty-aware AI systems.

- *Model Complexity*

The increasing complexity of deep learning models poses significant challenges for uncertainty estimation. Modern deep learning models often have millions of parameters and intricate architectures, making uncertainty estimation in such models non-trivial. Traditional uncertainty estimation methods may struggle to scale or converge effectively in the face of model complexity, limiting their practical utility in real-world applications.

- *Robustness and Generalization*

Ensuring the robustness and generalization capabilities of uncertainty estimation methods is another significant challenge. Uncertainty estimation methods should be robust to adversarial attacks, distributional shifts, and out-of-distribution samples to provide reliable uncertainty estimates in diverse environments and conditions. However, achieving robustness and generalization in uncertainty estimation is non-trivial and requires careful consideration of model assumptions and training procedures.

- *Relevance of Challenges*

These challenges are highly relevant as they directly impact the feasibility, usability and reliability of uncertainty estimation methods in real-world applications. Addressing these challenges is crucial for unlocking the full potential of uncertainty-aware AI systems and enabling better decision-making in uncertain and complex environments. Overcoming these challenges requires interdisciplinary collaboration and innovative research efforts across machine learning, statistics, and computational science.

### B. Future Directions of Uncertainty Estimation in Deep Learning

- *Scalable Methods*

Developing scalable uncertainty estimation methods that can handle large-scale datasets and complex models efficiently is essential for practical deployment in real-world applications. Scalable methods should leverage computational advances, such as parallelization, distributed computing, and hardware acceleration, to accelerate uncertainty estimation without sacrificing accuracy or reliability.

- *Interpretable Methods*

Enhancing the interpretability of uncertainty estimates is crucial for fostering trust and usability in uncertainty-aware AI systems. Future research should focus on developing interpretable uncertainty estimation methods that provide intuitive interpretations of uncertainty and enable end-users



to make informed decisions based on uncertainty-aware predictions.

- *Data-Efficient Methods*

Developing data-efficient uncertainty estimation methods that can leverage limited labeled data effectively is essential for expanding the reach of uncertainty-aware AI systems to diverse domains. Data-efficient methods should leverage techniques such as transfer learning, semi-supervised learning, and active learning to leverage unlabeled data and domain knowledge effectively for uncertainty estimation.

- *Deep Integration*

Integrating uncertainty estimation into the training, inference, and decision-making processes of deep learning models is essential for realizing the full potential of uncertainty-aware AI systems. Deep integration involves designing end-to-end pipelines that seamlessly incorporate uncertainty estimation into the model development lifecycle, enabling robust and reliable predictions in real-world applications.

- *Robustness and Generalization*

Improving the robustness and generalization capabilities of uncertainty estimation methods is essential for ensuring the reliability and effectiveness of uncertainty-aware AI systems across diverse environments and conditions. Future research should focus on developing robust uncertainty estimation methods that can handle adversarial attacks, distributional shifts, and out-of-distribution samples effectively.

## VII. LIMITATIONS

Uncertainty estimation in deep learning is a critical aspect of building robust and reliable artificial intelligence (AI) systems. It provides valuable insights into the confidence and reliability of model predictions, enabling better decision-making in various applications. However, uncertainty estimation also faces several limitations that need to be addressed for its effective implementation. In this comprehensive overview, we will delve into the limitations of uncertainty estimation in deep learning, discuss the challenges and drawbacks associated with estimating uncertainty in deep learning models, and explore potential solutions and areas of future research in this field.

### A. Data Scarcity

One of the primary limitations of uncertainty estimation in deep learning is data scarcity. Many uncertainty estimation methods require large amounts of labeled data to quantify uncertainty accurately. However, in many real-world scenarios, collecting labeled data for uncertainty estimation can be costly, time-consuming, or impractical. Moreover, labeled data may not always be available or representative of the true data distribution, leading to biased or unreliable uncertainty estimates.

### B. Model Complexity

The increasing complexity of deep learning models poses significant challenges for uncertainty estimation. Modern deep learning models often have millions of parameters and intricate architectures, making uncertainty estimation in such models non-trivial. Traditional uncertainty estimation methods may struggle to scale or converge effectively in

the face of model complexity, limiting their practical utility in real-world applications.

### C. Interpretability

Interpreting uncertainty estimates and communicating them effectively to end-users pose significant challenges in uncertainty estimation. Uncertainty estimates often lack intuitive interpretations, making it difficult for non-experts to understand and trust AI-driven systems. Moreover, visualizing uncertainty estimates in a meaningful and interpretable manner is non-trivial, further complicating the task of conveying uncertainty information to end-users.

### D. Evaluation Metrics

Evaluating the performance of uncertainty estimation methods is challenging due to the lack of standardized evaluation metrics. Unlike traditional classification or regression tasks, where metrics such as accuracy or mean squared error can be used to evaluate model performance, uncertainty estimation does not have universally accepted evaluation metrics. This makes it difficult to compare the performance of different uncertainty estimation methods and assess their effectiveness in real-world applications.

### E. Calibration

Ensuring the calibration of uncertainty estimates is another significant challenge in deep learning. Calibrated uncertainty estimates should accurately reflect the true uncertainty inherent in the data and model predictions. However, achieving calibration in uncertainty estimation is non-trivial and requires careful calibration of model outputs to match the true distribution of uncertainty. In many cases, uncertainty estimates may be mis-calibrated, leading to overconfident or underconfident predictions.

## VIII. POTENTIAL SOLUTIONS AND AREAS OF FUTURE RESEARCH

### A. Data Augmentation

One potential solution to address data scarcity in uncertainty estimation is data augmentation. Data augmentation techniques, such as rotation, translation, and scaling, can generate synthetic data samples to augment the training dataset, allowing uncertainty estimation methods to leverage more diverse and representative data for training.

### B. Model Simplification

Simplifying deep learning models by reducing model complexity or using simpler architectures can help mitigate the challenges associated with uncertainty estimation. Simplified models may be more amenable to uncertainty estimation methods and require fewer computational resources, making uncertainty estimation more practical and efficient.

### C. Interpretable Uncertainty Estimates

Developing interpretable uncertainty estimation methods that provide intuitive interpretations of uncertainty can enhance the usability and trustworthiness of uncertainty-aware AI systems. Interpretable uncertainty

estimates enable end-users to understand and interpret uncertainty information effectively, facilitating better decision-making in real-world applications.

#### D. Standardized Evaluation Metrics

Establishing standardized evaluation metrics for uncertainty estimation is essential for benchmarking and comparing the performance of different uncertainty estimation methods. Standardized metrics can provide a common framework for evaluating uncertainty estimation methods and assessing their effectiveness in various applications, facilitating advances in uncertainty estimation research.

#### E. Calibration Techniques

Developing calibration techniques for uncertainty estimation methods is crucial for ensuring the reliability and accuracy of uncertainty estimates. Calibration techniques adjust the outputs of uncertainty estimation methods to match the true distribution of uncertainty, improving the calibration of uncertainty estimates and enhancing the reliability of AI-driven systems.

### IX. CONCLUSION

In conclusion, uncertainty estimation in deep learning holds immense promise for enhancing the reliability, interpretability, and ethical implications of AI systems. By accurately quantifying uncertainty, we can improve decision-making processes, mitigate risks, and foster trust in AI-driven solutions. However, addressing the challenges associated with uncertainty estimation requires concerted efforts from researchers, policymakers, and practitioners. Future research directions may include developing scalable and interpretable uncertainty estimation methods, investigating the societal impact of uncertainty-aware AI systems, and integrating uncertainty estimation into regulatory frameworks for responsible AI deployment. Ultimately, uncertainty estimation stands as a cornerstone in the quest for trustworthy and resilient AI systems in an increasingly complex and uncertain world.

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